STAT 757– HW #2

1. Suppose random vector $\mathbf{X} = (X_1, X_2, X_3)$ is a random sample of three observatons from a discrete uniform distribution with sample space $S = \{1, 2, 3\}$. Describe the set of outcomes (i.e., write out the full set of one or more triplets) that corresponds to the event

$$\sum_{i=1}^{3} \mathbf{X_i} = 5.$$

2. Suppose events A and B have $P(A) = 0.2$ and $P(B) = 0.25$. What is the probability of A or B occurring if the two events are mutually exclusive? If $P(A \cup B) = 0.4$ are A and B mutually exclusive and/or independent? If C is the complement of $A \cup B$, what is $P(C)$?

3. Suppose you draw 3 cards from a standard deck of 52, and all three are Aces. What is the probability of the fourth card also being an Ace, i.e., what is P(4th an Ace | 3 Aces)? Answer this question (1) from a counting-based argument, and (2) using the definition of conditional probability.

4. Suppose X_1,...,X_n are independent exponentially distributed random variables each with mean $1/r$. By definition, that means random variable $Y = X_1 + \ldots + X_n$ is Gamma distributed with rate r and shape n. Calculate $E(Y)$ using properties of expectation to confirm that the mean of such a gamma distribution is n/r.

5. Suppose the sample space for r.v. X is the unit interval [0,1], and X has density function $f(x) = 4x^3$. Find the cdf of X. Then use it to calculate $P(X \in [1/2, 1])$.

6. Compute the coefficient of variation (CV) for a Normal random variable with mean $\mu = 18$ and variance $\sigma^2 = 9$.

7. What is the difference between the Strong Law of Large Numbers, the Weak Law of Large Numbers, and the Central Limit Theorem?

8. Suppose continous r.v.s X_1, X_2, ..., X_k are *iid* with density function $f(x)$. Suppose $\mathbf{X}$ is the random vector $(X_1, \ldots, X_k)$. What is the joint density function of $\mathbf{X}$?

9. Suppose there are 15 individuals in a population of 100 who carry a disease. Suppose you randomly select 50 of them, and test them for the disease, and let r.v. X be the number that actually have it (suppose a 100% accurate test). What named distribution describes the distribution of X?