Week 3 – Wednesday

Mathematical Modeling (Math 420/620)

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9 Sept, 2015

NSF Funded Undergraduate Workshop on Integrating Dynamics and Stochastics:

The Division of Applied Mathematics at Brown University invites undergraduate students to attend a workshop on Integrating Dynamics and Stochastics on **November 13, 2015**. The goal of the workshop is to inform advanced undergraduate students who are **interested in pursuing graduate studies in research areas at the intersection of dynamics and stochastics**. The workshop will feature talks by faculty, postdocs and graduate students working in various fields related to dynamics and stochastics. There will also be opportunities for attendees to interact with faculty, postdocs and students in the division. The workshop will run from 9am to 5pm with social activities following the workshop.

To apply, please see the following `mathprograms.org` page:

`http://www.mathprograms.org/db/programs/384`

For additional information, see:

`http://www.dam.brown.edu/people/lipshutz/workshop2015.html`

Projects

Two Approaches:

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A: Not trivially easy, not a full publication (perhaps a start of one?).

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A: Yes, but only on closely relate *independent* projects.
Q: ??

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Update on your project status **due on Monday**.
Which journals? Which kinds of questions?

Example 1.1

Q: When should the pig be sold *to maximize profit*?

Profit P is revenue (R) minus cost (C), therefore

$$\begin{aligned} P(t) &= R(t) - C(t) \\ &= p \cdot w - c \cdot t \\ &= (0.65 - 0.01\ t)(200 + 5\ t) - 0.45\ t \end{aligned}$$

Example 1.1: Generalized Model

Profit Equation:

$$P(t) = pw - ct = \underbrace{\overbrace{(p_0 - rt)}^{p(t)}\overbrace{(w_0 + gt)}^{w(t)}}_{\text{Revenue}} - \underbrace{ct}_{\text{Cost}}$$

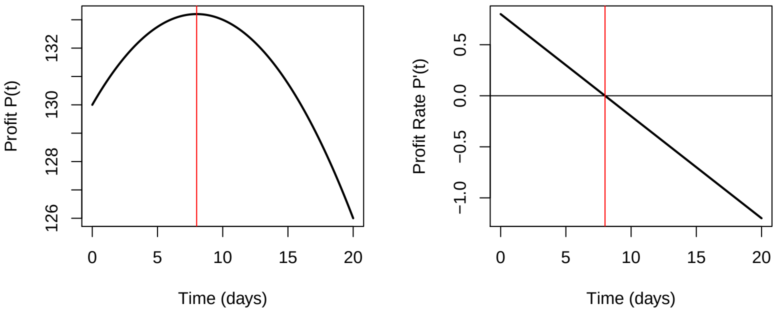
Variables (time dependent):

- p - pig price per pound
- w - weight (lbs)
- t - time (days)

Parameters (constants):

- $p0$ - initial pig price per pound
- r - daily decrease in price p
- $w0$ - initial weight of pig
- g - daily growth rate of pig
- c - cost to keep pig ($/day)

```
## R code to plot P(t) and P'(t)
x=seq(0,20,length=200)
Pt=expression((0.65-0.01*x)*(200+5*x)-0.45*x,'x')
dPt=D(Pt,'x')
plot(x, (0.65-0.01*x)*(200+5*x)-0.45*x, type="l", lwd=2,
        ylab="Profit P(t)", xlab="Time (days)"); abline(v=8, col="red")
plot(x,eval(dPt),type="l",lwd=2,ylab="Profit Rate P'(t)",xlab="Time (days)")
xOpt = 8;
abline(h=0); abline(v=xOpt, col="red")
```



```
## R code to plot P(t) and P'(t)
x=seq(0,20,length=200)
P=function(x) { (0.65-0.01*x)*(200+5*x)-0.45*x };
dP=function(x){ (8-x)/10 }; # Use Maxima!
plot(x, P(x), type="l", lwd=2,
        ylab="Profit P(t)", xlab="Time (days)"); abline(v=8, col="red")
plot(x, dP(x),type="l",lwd=2,ylab="Profit Rate P'(t)",xlab="Time (days)")
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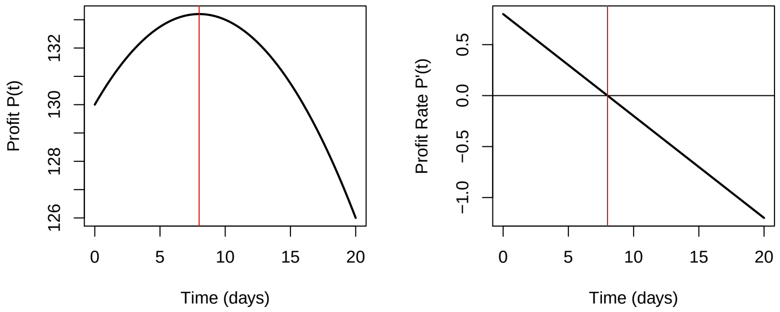

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plot(x, dP(x),type="l",lwd=2,ylab="Profit Rate P'(t)",xlab="Time (days)")
xOpt = optimize(P,c(0,20),maximum=TRUE) # 1D optimization in R
abline(h=0); abline(v=xOpt, col="red")
```

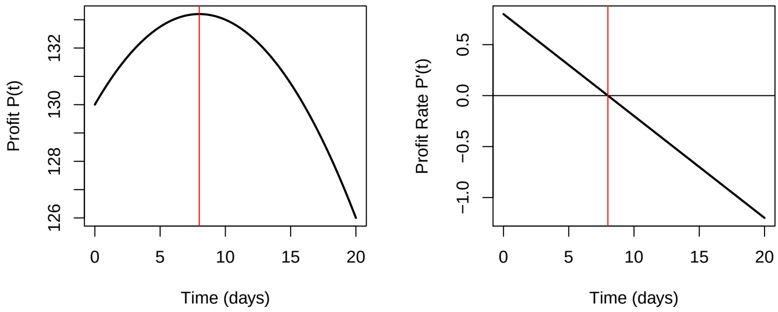

Multivariable Optimization: Ex 2.1 (Pg 21)

Q: How many 19- & 21-inch TVs maximize profit?

Profit Equation:

$$P(x_{19}, x_{21}) = p\,x_{19} + q\,x_{21} - C = (p_0 - p_{19}\,x_{19} - p_{21}\,x_{21})x_{19} + (q_0 - q_{19}\,x_{19} - q_{21}\,x_{21})x_{21} - (c_0 + c_{19}\,x_{19} + c_{21}\,x_{21})$$

Variables (time dependent):

x_{19} - number of 19in TVs sold

x_{21} - number of 21in TVs sold

p - 19in TV selling price ($)

q - 21in TV selling price ($)

Parameters (constants):

$p0$ - Retail price of 19-inch TV

$q0$ - Retail price of 19-inch TV

p_{19} - 19in discount / 19in TV sold

p_{21} - 19in discount / 21in TV sold

q_{19} - 21in discount / 19in TV sold

...

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Using Maxima, we can find and solve

$$\begin{aligned}\frac{\partial P}{\partial x_{19}} &= p_0 - 2p_{19}x_{19} - (q_{19} + p_{21})x_{21} - c_{19} = 0 \\ \frac{\partial P}{\partial x_{21}} &= q_0 - (q_{19} + p_{21})x_{19} - 2q_{21}x_{21} - c_{21} = 0\end{aligned}$$

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Solving and using the given parameter values yields

$$\mathbf{x_{19} = 4735.04}\ldots \qquad \mathbf{x_{21} = 7042.74}\ldots$$

Multivariable Optimization: Ex 2.1 (Pg 21)

Approach #2: Use generic optimization routines to computationally maximize $P(x_{19}, x_{21})$ over $x_{19}, x_{21} > 0$.

See Ch2-optimization.R