Exam Topics: Overview

Math 420/620 – Fall 2015

IN-CLASS EXAM

I. Modeling Approach -- See Week 1 & 2 notes/slides

a. Familiarity with the "Five-step Method" **as treated in class**, not just in the textbook.
   i. It wouldn't hurt to know how to *Lie*, *Cheat*, and *Steal*.

II. Optimization

a. One-variable Optimization – Analytical Approaches
b. Multi-variable Optimization – Analytical Approaches
c. LaGrange Multipliers for Constrained Optimization
   i. Know how to use KKT Multipliers if given the equations
d. Sensitivity Analysis & Relative Sensitivities (see Week 2 M slides)

III. Dynamical Systems (ODEs)

a. Mean Field equations
   i. Linear terms & exponential rates of transitions between states
   ii. Limit of discrete time (mean field) models
b. Terminology: State Variables, Initial Conditions, State Space, etc.
c. Equilibria
   i. Definition, know how to find them, etc.
d. Stability Concepts
e. Local Asymptotical Stability (LAS)
   i. LAS for 1D models:
      1. Find equilibrium, determine stability using "phase portraits"
   ii. LAS (General): Jacobian, Eigenvalues.
   iii. Routh-Hurwitz Criteria (if they would be provided to you; could you use them?)
f. "Phase Space" plots ("phase portraits")
   i. 1D models (x vs dx/dt=f(x) plot to determine equilibria & stability)
      1. Sign of f'(x*) as a stability criterion
   ii. 2D models (x vs y vector field),
      1. Find and plot nullclines; use them to draw vector field
      2. Given a vector field, nullclines, trace out an approximate solution curve
g. Explicit Solutions to ODEs
   i. Integration by Parts (Ex: Logistic Equation)
   ii. What is the solution to $dx/dt = r\,x$, $x(0)=x0$?

h. Discrete-time Dynamical Systems (Maps)

   i. Equilbria & Stability Condition basics (**see Nov. 9 notes**)

IV. Probabilistic & Stochastic Models

a. Basic Probability Calculus
   i. Calculate probabilities given a PDF/PMF or a CDF
   ii. Compute expected values (means, variances, E(g(x)), etc)
   iii. Conditional Probability
b. Probability Distributions
   i. Know when to use the distributions in the "prob-distributions.pdf" table
   ii. Relationships between distributions (e.g., Geometric vs Negative Binomial)
   iii. CLT and LLN
c. Poisson Process
d. Markov Chains
   i. Transition Matrices
   ii. Stationary Distributions
   iii. Irreducibility
   iv. Transient & Recurrent states

V. Other

a. The exam assumes you know some basic mathematical concepts!
   i. Pre-Requisites: The highlights from Calculus I-III, Probability, etc.
   ii. Basic Linear Algebra used in class: Ex: Eigenvalue and Eigenvector definition, matrix multiplication etc.

Additonal Topics for TAKE-HOME EXAM

I. **Q:** Could you modify code provided in class (or in HW solutions) to tackle similar problems?
   a. This includes R and wxMaxima files.
   b. You will be allowed to use other software (e.g. python) if you'd prefer, BUT this may limit partial credit opportunities if I can't read/interpret electronic supplementary files.

II. **Q:** Would you know how to use R's help documentation (or find examples/tutorials online) to implement a new R function to tackle a new method of analysis?

III. R basics, e.g., defining functions, using packages, reading/writing files (e.g. data), for loops

IV. Graphics: plot(), points(), curve(), abline(), etc. [or equivalent in similar software]

V. Optimization using optimize(), optim() or optimx()

VI. Eigenvalues and eigenvectors in R or perhaps Maxima [or equivalent in similar software]

VII. Numerical solutions of ODEs using ode() from the deSolve package in R

VIII. Simulation of Probabilistic & Stochastic Models using R
   a. Poisson Process (Homogeneous)
   b. Random sample, Quantile, Density/Mass, CDF functions in R for named distributions

IX. Matrix operations in R

***** BONUS POINTS MAY BE AWARDED FOR WRITING UP THE TAKE-HOME EXAM USING LATEX! *****