MATH XXX – Spring 2026 – Homework #?? NAME-HERE

Instructions: Please complete the following exercises, etc.

Exercises:

1. (Adapted from Wiggins, §18.7 exercise 4) For each of the following systems, do a center manifold reduction at the origin (with the bifurcation occurring at $\epsilon = 0$) and use the equation for the center manifold dynamics to classify the bifurcation type.

   (a) Suppose x is on S (the circle, 0 to 2π) and $v \in \mathbb{R}$.

   $$\begin{aligned}\frac{dx}{dt} &= -x + \epsilon v + v^2 \\ \frac{dv}{dt} &= -\sin(x)\end{aligned}$$

   (b) Suppose x and y are real, and

   $$\begin{aligned}\frac{dx}{dt} &= \frac{x}{2} + y + x^2 y \\ \frac{dy}{dt} &= x + 2y + \epsilon y + y^2\end{aligned}$$

   Solution:
   (a)
   (b)

2. Recall that $\frac{d}{dx}\tan(x) = \sec^2(x)$. For $a, b \in (-\pi/2, \pi/2)$, find

   $$\int_a^b \sec^2(u)\, du.$$

   Solution: