

Instructions: Please complete the following exercises, etc.

Exercises:

1. (Adapted from Wiggins, §18.7 exercise 4) For each of the following systems, do a center manifold reduction at the origin (with the bifurcation occurring at $\epsilon = 0$) and use the equation for the center manifold dynamics to classify the bifurcation type.

a) Suppose x is on S (the circle, 0 to 2π) and $v \in \mathbb{R}$.

$$\begin{aligned}\frac{dx}{dt} &= -x + \epsilon v + v^2 \\ \frac{dv}{dt} &= -\sin(x)\end{aligned}$$

b) Suppose x and y are real, and

$$\begin{aligned}\frac{dx}{dt} &= \frac{x}{2} + y + x^2y \\ \frac{dy}{dt} &= x + 2y + \epsilon y + y^2\end{aligned}$$

Solution:

a)
b)

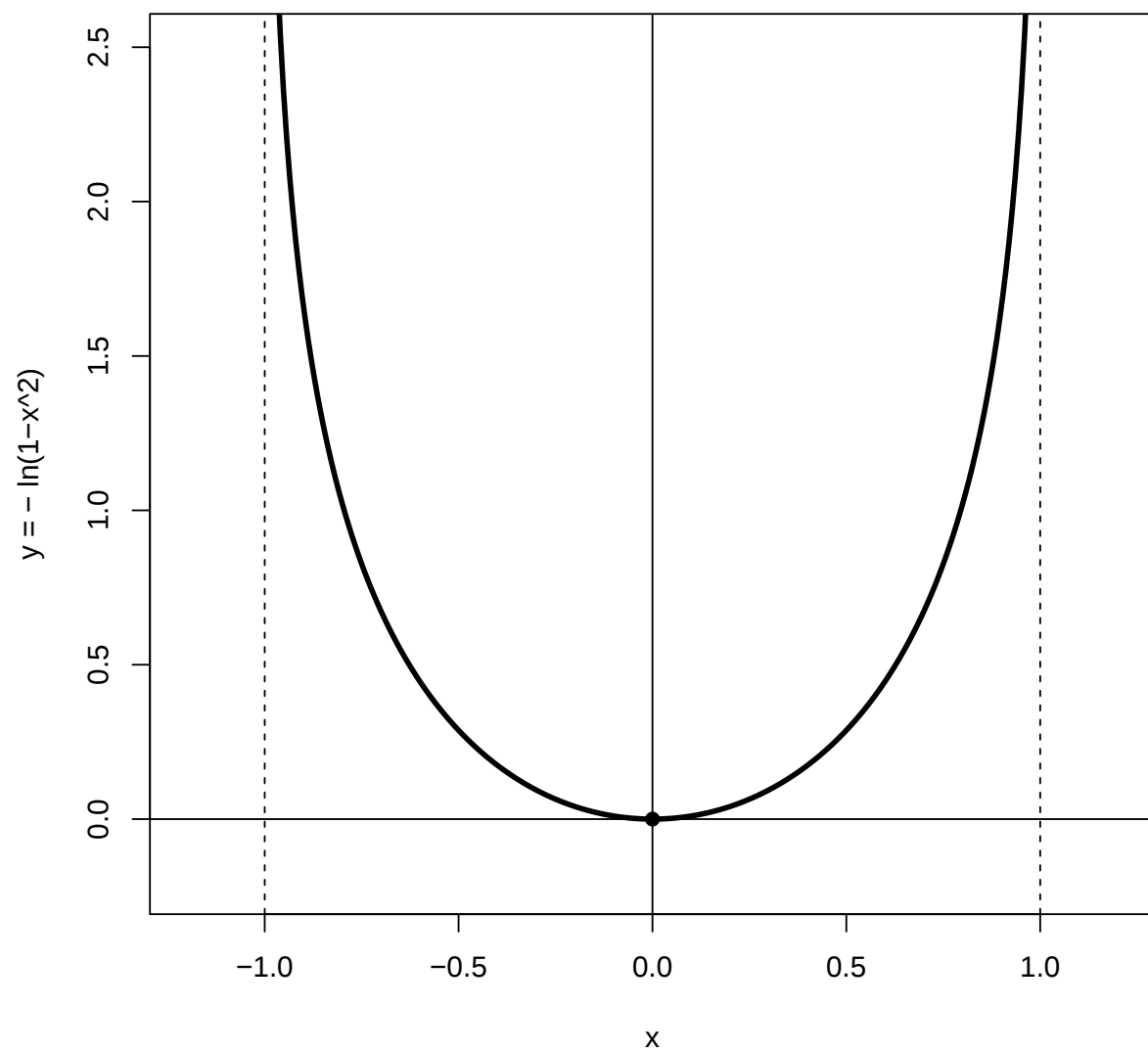
2. Recall that $\frac{d}{dx} \tan(x) = \sec^2(x)$. For $a, b \in (-\pi/2, \pi/2)$, find

$$\int_a^b \sec^2(u) du.$$

Solution:

3. You can embed snippets of R code (or python code, etc.) into a LaTeX document using the knitr package in R to parse the code and format the code and output in a LaTeX document. For more, see <https://pauljhurtado.com/latex/#quickrknitr>. If you're familiar with R Markdown code chunks, these LaTeX code chunks follow the same general format with only a few minor differences! :)

```
## R code to draw the graph below
curve(-log(1-x^2), -1, 1, n=250, ylim=c(-0.2, 2.5), lwd=3,
ylab="y = - ln(1-x^2)", xlim=c(-1.2, 1.2))
points(0, 0, pch=19)
abline(h=0, v=0)
abline(v=1, lty=2)
abline(v=-1, lty=2)
```



```
## Example R code and output
1+1
## [1] 2
diff(c(1,2,5,10))
## [1] 1 3 5
```