Stochastic Models
Week 11 – Monday

Mathematical Modeling (Math 420/620)

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Math & Stat Colloquium

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Homogeneous Poisson Process

Times between events are exponential (rate r):

$$T_k \sim \text{Exponential}(r)$$

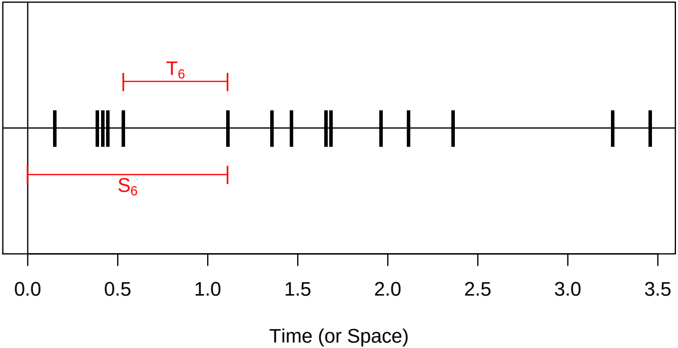
Event times are Gamma (shape k, rate r), i.e. $S_k = \sum_{i=0}^{k}$:

$$S_k \sim \text{Gamma}(k, r)$$

Number in interval $[0, T]$ is as a counting process N_T which is **Poisson** distributed with mean $\lambda = r\,T$, i.e.

$$P(N_T = n) = \exp(-\lambda)\,\frac{\lambda^n}{n!}$$

Homogeneous Poisson Process




```
Tk = rexp(15,rate=5);   Sk = cumsum(Tk);
plot(Sk,Sk*0,pch="|",xlab="Time (or Space)",yaxt="n", xlim=c(0,max(Sk)));
```

Inhomogeneous Poisson Process

Assumes **rate** r **non-constant**, for example

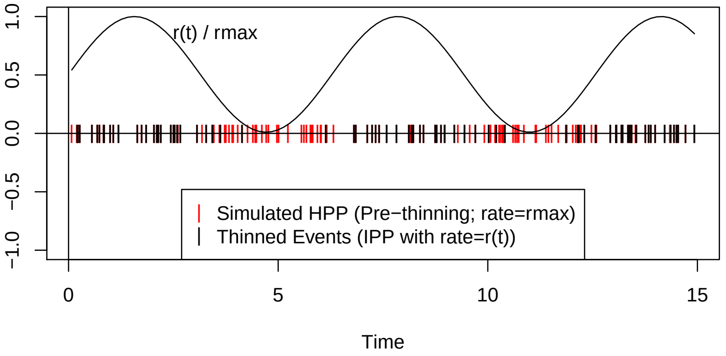
$$r(t) = r_0 + A\, sin(\omega\, t)$$

The **Counting Process** has mean

$$E(N_T) = \frac{1}{T} \int_0^T \lambda(t)\, dt = \int_0^T r(t)\, dt$$

where we define $\lambda(t) = r(t)\, T$.

Inhomogeneous Poisson Process



Inhomogeneous Poisson Process (Simulation in R)

```
# First the rate function r(t) = r0 + A sin(w t)
r0 = 5;  A=4.9;  w=1;  rmax = r0+A;
r = function(t) { r0 + A*sin(w*t) }

# Simulate at rmax=r0+A
set.seed(1)
Tk = rexp(150,rate=rmax);   Sk = cumsum(Tk);

# Thin the events generated at rmax w.p. r(t)/rmax
Ps = r(Sk)/rmax;      RNs = runif(length(Sk))
keep = (RNs <= Ps);   # Element-wise comparison. Returns vector of T/F.
Skthinned = Sk[keep]; # Keep only those with RNs <= Ps

# Plot thinned and remaining events.
plot(Sk,0*Sk,pch="|",col="red", xlab="Time", ylab=""); abline(v=0,h=0)
points(Skthinned,0*Skthinned,pch="|")
# Plot a normalized r(t) curve
curve(r(x)/rmax, add=TRUE); text(3,0.85,"r(t) / rmax")
legend("bottom",c("Simulated HPP (Pre-thinning; rate=rmax)",
   "Thinned Events (IPP with rate=r(t))"), col=c("red","black"),pch='|')
```